ELASTIC DEFORMATION OF A GRAINY MEDIUM SATURATED BY LIQUID

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1. The problem of elastic deformation of an inhomogeneous medium is of great practical importance. It is known that the structure of a medium has a significant effect on deformation processes: the size and form of inhomogeneities, composition and physical properties of inclusions, etc. [1]. A saturated granular medium is an idealized model of a two-phase soil, which permits some consideration of structure. In [2], four types of elastic spheres, differing in size, were used to model the granular medium. Packing was assumed to be arbitrary. A contribution to deformation was considered only from the contact regions of spheres of one kind. An expression for the change in volume of a region including N spheres and liquid was obtained from energy considerations. A model consisting of ordered equal spheres was used in [3] to calculate porosity and the filtration coefficient, and in [4] to calculate the pressure on a fixed support wall. Effective moduli were determined for elastic spheres immersed in an elastic liquid in [5]. Cubical packing was used with assumption of absence of stress in the horizontal plane. The question under consideration has recently found an application in transmission of chemical materials through granular catalysts [6]. In [7] a generalized approach was developed for description of deformation of a saturated porous medium with consideration of filtration flows. However, interphase interaction was considered by another method.

The present study will calculate the solid phase stress tensor as a function of liquid pressure in the pores and external (mineral) pressure. The nonlinear character of deformation of an elastic granular medium will be demonstrated.

1. We will consider a volume V of a granular medium saturated by a liquid. Let the elastic grains of spherical form be equal in size and arranged in an arbitrary manner. These grains are submerged in a compressible viscous liquid. Figure 1 shows a portion of the volume V including one sphere and contact regions.

We call attention to the fact that the medium under study here has certain unique features which distinguish it from other structure in the class of heterogeneous media [7].

1. Continuity of both component phases. As a consequence, it is possible for the liquid pressure p and the stress Γ_{ij} applied only to the solid phase (mineral pressure) through the contact regions to change independently.

2. Effect of the medium structure on the amount of deformation of the component phases. Because of this the value of the mean stress in the solid phase is not equal to the external Γ_{ij}. The difference is determined by interaction between the phases as well as the structure of the interphase surface. Similarly, for the liquid the change in volume of the pore space upon deformation does not correspond exactly to the change in volume of liquid in the pores, which leads to either a discontinuity in the mean stresses in the phases, or to flow of liquid into the adjacent elementary volume V. In reality, the pore space is limited by the interphase boundary S_*, which deforms together with the change in the volume V. On the surface S_* the stresses are always equal:

$$-p\delta_{ij} = T_{ij}|_{S_*},$$

where T_{ij} is the solid phase stress tensor. Let the deformation of the solid phase obey Hooke's law $\theta_1 = \sigma[3E(1-2\nu)]^{-1}$ ($\sigma = T_{11} + T_{22} + T_{33}$ is the sum of the normal stresses, is the Poisson coefficient, and E is the modulus of elasticity), while the liquid deformation obeys Tate's equation

$$\theta_2 = (1/a)\ln|p/p_0 + 1| \tag{1.1}$$

(θ is the relative change in volume). If V_0 is the volume before deformation, then in the general case $V = V_0(1-\theta)$. For the quantities θ and ρ we have the relationship $\theta = 1 - \rho_0/\rho$ (where V_0, ρ_0, p_0, a are constants). Hence we find the relationship between θ_1, θ_2 on the

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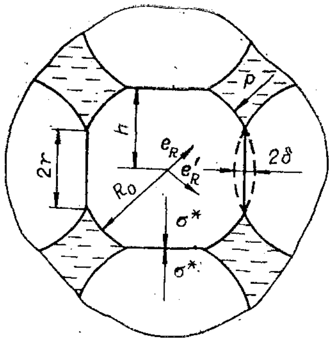

Fig. 1

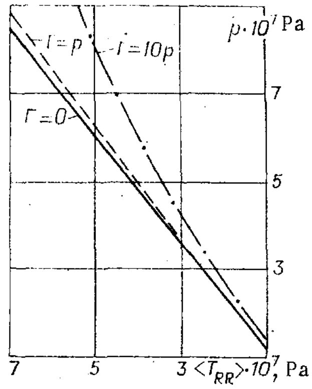

Fig. 2

boundary S_*:

$$\theta_1 = \frac{P_0}{E(1-2\nu)}(\exp\{a\theta_2\} - 1).$$

However, the liquid deforms due to the change in the total volume V and the change in volume of the grains. This fact does not allow exact calculation of the porosity under deformation. In [2, 8] attempts were made to calculate the change in porosity Δm, but these were very approximate. For example, in [8] it was assumed that $\theta_1 = \theta_2$, which can only occur in the special case in which the compressibilities of the phases are close in value.

We will obtain relationships applicable to the most general case of deformation of a saturated granular medium. From the additiveness of masses of elements occupying the volume V ($M = M_1 + M_2$) and the additiveness of the change in volumes after deformation ($\Delta V = \Delta V_1 + \Delta V_2$) it follows that

$$\rho = \rho_1(1-m) + \rho_2 m,\ \theta = \theta_1(1-m_0) + \theta_2 m_0.$$

Substituting in Eq. (1.2) $m_0 = V_{02}/V_0$, and expressions relating the quantities V and θ, we can prove the validity of Eq. (1.2).

We note that by choosing unordered orientation of the grains, we lose the possibility of exact calculation of the porosity itself. However this does not disallow assumption of the existence of some definite order around each individual particle. This can be justified by data from statistical analysis of the locations of contact areas. Thus, the choice of symmetrical disposition of the contact surfaces on a grain serves only to make calculations convenient. It is known that in deformation of the medium under study here either conservation of the masses of the solid and liquid component phases within a unit volume or loss of such conservation may occur [7]. We will call the first case deformation of the first sort. It is obvious that this case can be realized only at a deformation rate such that no overflow of liquid can occur. Then, as was noted earlier, stress quality cannot be established in the phases. When stress equality does exist, the value of the porosity must change in a completely defined manner. Expressing the quantities ρ in terms of θ and eliminating θ with the aid of Eq. (1.2), after transformations we obtain

$$m = m_0(1-\theta_2)/[1-\theta_1 + m_0(\theta_1-\theta_2)].$$

In the absence of deformations $\theta_1 = \theta_2 = 0$, $m = m_0$. We will term deformations of the second kind conditions such that there are overflows of liquid on the scale of the volume V, i.e., relative displacement of phases occurs, and due to liquid filtration tangent stresses appear on the interphase boundary S_*. This is accompanied by loss of conservation of phase mass within the volume V. We note that for conservation of the solid phase overpacking of the particles must not exist. As before, Eq. (1.2) is valid.

2. Let the deformation conditions correspond to a deformation of the first kind. In addition, tangent stresses are absent at the contacts. This can be achieved if before deformation the grains are able to move relative to each other. And the value of the deformation and state of the surface create conditions for the absence of significant tangent stresses. These conditions permit us to write $\Gamma_{ij} = \Gamma_*\delta_{ij}$. The grains are elastic equal-sized spheres, packed in a manner such that the close order is close to a cubical structure. We describe the deformation of the contacting spheres with the results of the Hertz problem. We recall

that the radius of the contact surface r and the degree of warping of the spheres δ (see Fig. 1), we have [9]

$$r=\left(\frac{3}{4}Q\frac{\nu-1}{G}R_0\right)^{1/3},\quad \delta=\left(\frac{3}{4}Q\frac{\nu-1}{G}R_0^{-1/2}\right)^{2/3}, \tag{2.1}$$

where G is the modulus of elasticity; $Q = 2\pi r^2\sigma^*$; σ^* is the stress at the contact, or according to [7], the effective stress. In the simplest possible case we write $\sigma^* = \Gamma_* - p$. Considering the relationship of the quantities ν, G to Young's modulus E, we transfer Eq. (2.1) to the form

$$r=\frac{4}{3}\pi\frac{\sigma^*}{E}R_0,\quad \delta=\left(\frac{4}{3}\pi\frac{\sigma^*}{E}\right)^2 R_0,$$

where R_0 is the sphere radius.

Thus, due to the idealization of the medium used, the value of the solid phase deformation can be calculated by averaging the stress tensor over the volume of a sphere. We perform this averaging with the aid of the expression

$$\langle T_{ij}\rangle=\frac{3}{4\pi R_0^3}\int_0^{R_0}dR\int_0^{2\pi}d\lambda\int_0^{\pi}T_{ij}R^2\sin^2\varphi d\varphi. \tag{2.2}$$

We will note that partial spatial averaging of the structure was performed by transformation from inhomogeneous particles to spherically formed particles of equal size. Thus, in Eq. (2.2) on the left we have the completely averaged expression for the tensor components $\langle T_{ij}\rangle$. The components of the tensor T_{ij} in the integrand can be found by solving the boundary problem for deformation of an elastic medium with piecewise-inhomogeneous loading, as shown in Fig. 1. We will use the known solution of this problem presented in [9]. The condition of absence of tangent stresses permits writing an expression for the radial vector $\mathbf{P}_R$ within the sphere in the form

$$\mathbf{P}_R=\frac{1}{4\pi}\sum_{n=1}^{\infty}(2n+1)\left(\frac{R}{R_0}\right)^{n-1}\int_0^{2\pi}d\lambda'\int_0^{\pi}d\varphi'\mathbf{P}_R^0(\lambda',\varphi')P_n(\gamma)+\frac{1}{4\pi}\sum_{n=3}^{\infty}\frac{(2n+1)(2n-2)}{2[n^2-(1-2\nu)n+1-\nu]}\left[\left(\frac{R}{R_0}\right)^{n-3}\right.$$
$$\left.-\left(\frac{R}{R_0}\right)^{n-1}\right]\int_0^{2\pi}d\lambda'\int_0^{\pi}d\varphi'\mathbf{P}_R^0(\lambda',\varphi')\left[IP'_{n-1}(\gamma)+\mathbf{e}_R\mathbf{e}_R P''_{n-2}(\gamma)+\mathbf{e}'_R\mathbf{e}'_R P''_n(\gamma)-\left(\mathbf{e}'_R\mathbf{e}_R+\mathbf{e}_R\mathbf{e}'_R\right)P''_{n-1}(\gamma)\right],$$
$$\gamma=\cos\varphi\cos\varphi'+\sin\varphi\sin\varphi'\cos(\lambda-\lambda'), \tag{2.3}$$

where φ', λ' are spherical coordinates permitting the definition of the boundary condition on the sphere surface; P_n, P'_{n-1}, P''_{n-2}, P''_{n-1}, P''_n are Legendre polynomials; I is a unit tensor; $\mathbf{e}_R$, $\mathbf{e}'_R$ are unit vectors. The expression for the sum of the main stresses [9] will be

$$\sigma=\frac{1+\nu}{4\pi}\sum_{n=1}^{\infty}\frac{4n^2-1}{n^2-(1-2\nu)n+1-\nu}\left(\frac{R}{R_0}\right)^{n-1}\int_0^{2\pi}d\lambda'\int_0^{\pi}d\varphi'\mathbf{P}_R^0(\lambda',\varphi')\left[\mathbf{e}'_R P'_n(\gamma)-\mathbf{e}_R P'_{n-1}(\gamma)\right]. \tag{2.4}$$

Thus the components T_{ij} will be completely defined by multiplying the left side of Eq. (2.3) by the unit vectors $\mathbf{e}_R, \mathbf{e}_\varphi, \mathbf{e}_\lambda$. It can easily be proved that only the product $\mathbf{e}_R \cdot \mathbf{P}_R = T_{RR}$ is nonzero. We will divide the limits of the double integral in Eq. (2.3) in accordance with the boundary condition

$$\int_0^{2\pi}d\lambda'\int_0^{\pi}d\varphi'=\int_0^{2\pi}d\lambda'\left[\int_{h/R_0}^{1}+\int_{-1}^{-h/R_0}\right]d(\cos\varphi')+\int_0^{2\pi}d\lambda'\left[\int_{r/R_0}^{h/R_0}+\int_{-h/R_0}^{-r/R_0}\right]$$
$$\times d(\cos\varphi')+\left[\int_0^{\operatorname{arctg} r/R_0}+\int_{\operatorname{arctg} h/r}^{\pi-\operatorname{arctg} h/r}+\int_{\pi-\operatorname{arctg} r/h}^{\pi+\operatorname{arctg} r/h}+\int_{\pi+\operatorname{arctg} h/r}^{2\pi-\operatorname{arctg} h/r}+\int_{2\pi-\operatorname{arctg} r/h}^{2\pi}\right]$$
$$\times d\lambda'\int_{-r/R_0}^{r/R_0}d(\cos\varphi')+\left[\int_{\operatorname{arctg} r/h}^{\operatorname{arctg} h/r}+\int_{\pi-\operatorname{arctg} h/r}^{\pi-\operatorname{arctg} r/h}+\int_{\pi+\operatorname{arctg} r/h}^{\pi+\operatorname{arctg} h/r}+\int_{2\pi-\operatorname{arctg} h/r}^{2\pi-\operatorname{arctg} r/h}\right]d\lambda'\int_{-r/R_0}^{r/R_0}d(\cos\varphi'). \tag{2.5}$$

Substituting Eq. (2.5) in Eqs. (2.3), (2.4), differentiating in the expressions for P_n, and maintaining at least six terms in the infinite series of Eq. (2.3), we write

$$T_{RR} = \frac{\sigma^*}{4\pi}\left\{7\left(1-\frac{h}{R_0}\right)+6\left(1-\frac{h^3}{R_0^3}\right)+\pi\left(\frac{2}{3}-\frac{h}{R_0}+\frac{h^3}{3R_0^3}\right)+6\frac{r}{R_0}+\frac{r^3}{R_0^3}+\frac{r}{R_0}\left(\pi+2\,\mathrm{arctg}\frac{r}{h}-2\,\mathrm{arctg}\frac{h}{r}\right)+\left(\frac{r}{R_0}-\frac{r^3}{3R_0^3}\right)\right.$$
$$\times\left.\left(\pi+2\,\mathrm{arctg}\frac{r}{h}-2\,\mathrm{arctg}\frac{h}{r}\right)\right\}+\frac{p}{4\pi}\left\{5\left(\frac{h}{R_0}-\frac{r}{R_0}\right)\right.$$
$$\left.+\left(\frac{h^3}{R_0^3}-\frac{r^3}{R_0^3}\right)+\pi\left(\frac{h}{R_0}-\frac{h^3}{3R_0^3}-\frac{r}{R_0}+\frac{r^3}{3R_0^3}\right)+2\frac{r}{R_0}\left(\mathrm{arctg}\frac{h}{r}-\mathrm{arctg}\frac{r}{h}\right)+2\left(7\frac{r}{R_0}+\frac{r^3}{R_0^3}-\frac{h^3}{3R_0^3}\right)\left(\mathrm{arctg}\frac{h}{r}-\mathrm{arctg}\frac{r}{h}\right)\right\}. \quad (2.6)$$

For the sum of the normal stresses

$$\sigma = \sigma^*\left[1-\frac{h}{R_0}+\frac{16}{9}\left(1-\frac{h^3}{R_0^3}\right)+\frac{16}{3}\left(\frac{2}{3}-\frac{h}{R_0}+\frac{h^3}{3R_0^3}\right)+4\frac{r}{R_0}+16\frac{r^3}{R_0^3}\right.$$
$$\left.+\frac{8}{5\pi}\left(\frac{r}{R_0}-\frac{r^3}{3R_0^3}\right)\right]\left(\pi+2\,\mathrm{arctg}\frac{r}{h}-2\,\mathrm{arctg}\frac{h}{r}\right)+\left[\frac{2}{3}\left(\frac{h}{r}-\frac{r}{h}\right)\right.$$
$$\left.+\frac{16}{9}\left(\frac{h^3}{R_0^3}-\frac{r^3}{R_0^3}\right)+\frac{16}{3}\left(\frac{h}{R_0}-\frac{h^3}{3R_0^3}\right)-\frac{r}{R_0}+\frac{r^3}{3R_0^3}+8\frac{r}{R_0}+\frac{32}{3}\frac{r^3}{R_0^3}+\frac{32}{\pi}\left(\frac{r}{R_0}-\frac{r^3}{3R_0^3}\right)\right]\left(\mathrm{arctg}\frac{h}{r}-\mathrm{arctg}\frac{r}{h}\right). \quad (2.7)$$

Then performing a number of operations including spatial averaging over the volume of a particle $(4/3)\pi R_0^3$ with the aid of Eq. (2.2), neglecting quantities in Eqs. (2.6), (2.7) small in comparison to the value $(\sigma^*/E)^2$ and considering the equality $\frac{r}{R_0}=\frac{4}{3}\pi\frac{\sigma^*}{E}$, we obtain simplified expressions

$$\langle T_{RR}\rangle = p\left(0.83+12.45\frac{p}{E}-12.45\frac{\Gamma_*}{E}\right); \quad (2.8)$$

$$\sigma = p\left(2.51+27.3\frac{p}{E}-27.3\frac{\Gamma_*}{E}\right). \quad (2.9)$$

3. Figure 2 shows the dependence of the radial component of the tensor $\langle T_{RR}\rangle$ on the quantities P, Γ_*. In the calculations, here and below we use the values $E = 3.5\cdot10^{10}$ Pa, $p_0 = 3.125\cdot10^8$ Pa, m = 0.2, which correspond to a sand medium, saturated with water. The relative contribution of these quantities to the component $\langle T_{RR}\rangle$ are based on choice of a boundary condition on the sphere surface, i.e., on the character of Eqs. (2.8), (2.9), obtained by spatial averaging of Eq. (2.2), and depends on the ratio between the contact area and the sphere surface. It is thus related to the structure of the medium. Comparison of Eqs. (2.8) and (2.9) allows us to write the approximate expression $\langle T_{RR}\rangle = \sigma/3$, which indicates the isotropic character of solid phase stress distribution. This result could be expected from the fact that no limitations were imposed on the relative location of the spheres within the volume. We note that the contributions of p and Γ_* to $\langle T_{RR}\rangle$ are not the same, and in the general case are nonlinear in character. The uncertainty of Eqs. (2.8), (2.9) is quite high, due to the poor convergence of the infinite series in Eq. (2.3). Substituting Eq. (2.9) in the expression for θ_1 and the quantities θ_1, θ_2 from Eq. (1.1) in Eq. (1.2), we obtain

$$\theta = \frac{p(1-m_0)}{E(1-2\nu)}\left(0.83+12.45\frac{p}{E}-12.45\frac{\Gamma_*}{E}\right)+\frac{m_0}{a}\ln\left|1+\frac{p}{p_0}\right|. \quad (3.1)$$

This is the change in total deformation of the volume V as a function of the stresses p and Γ_*, i.e., the generalized equation of state. The behavior of this function for the conditions selected is shown in Fig. 3. The curve $\theta(p, \Gamma_*)$ has a nonlinear character even for relatively monotonic elastic properties of the component phases. In [2] the dependence of the modulus of volume compression of the medium on the pressure p with an exponent of 1/6 was explained only by the sphere deformation conditions at the contact areas. However, it follows from Eq. (3.1) that the term describing the compressibility of the liquid phase also produces a contribution to the quantity θ. In other words, elastic compression of the given two-phase medium is caused by both deformation of the spheres in the contact region, i.e., the structure of the medium, and the compressibility of the liquid. In fact, in Eq. (1.1) the value of a for water is quite close to 1/6, which produces a dependence in Eq. (3.1) corresponding to the experimental data of [2].

Experimental values of parameters describing the elastic state of a saturated porous medium within the framework of the phenomenological Biot–Nikolaevskii model were presented in [10]. Using the results obtained, we calculate the compressibility parameter β (δ in the notation of [10]):

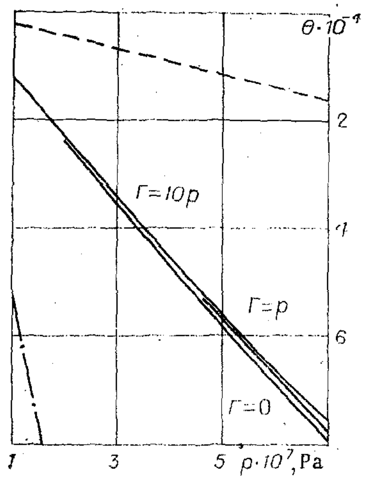

Fig. 3

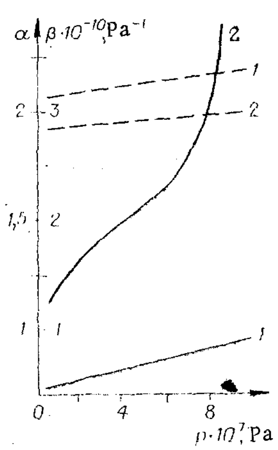

Fig. 4

$$\beta = \frac{\dfrac{1-m_0}{E(1-2\nu)}\left(0.83 + 12.45\,\dfrac{2p-\Gamma_*}{E}\right) + \dfrac{m_0}{p_0 a}\left(1 - \dfrac{p}{p_0}\right)^{-1}}{1 - \dfrac{(1-m_0)\,p}{(1-2\nu)\,E}\left(0.83 + 12.45\,\dfrac{p-\Gamma_*}{E}\right) - \dfrac{m_0}{a}\ln\left|1 + \dfrac{p}{p_0}\right|}. \quad (3.2)$$

Correspondingly, for the parameter α we have

$$\alpha = 3 - \frac{\Gamma_*}{p} + \frac{0.83E}{12.45p} + \frac{m_0(1-2\nu)E^2}{12.45(1-m_0)\,ap\left(1+\dfrac{p}{p_0}\right)p_0}. \quad (3.3)$$

These expressions are depicted in Fig. 4. The character of the dependence of the relative value of Eq. (3.3) on p corresponds qualitatively to experiment. Quantitative divergence is caused by the form of Eq. (2.8), i.e., the microstructure. In particular, the form of Eq. (2.8) does not permit analysis of deformation at $p = 0$.

The difference in the compressibility parameter as a function of pressure given by Eq. (3.2) from the results obtained in [10] is indicative of the simplification used in the present study. Under real conditions deformation of the medium under study is characterized in the initial stage by the effect of the inelastic component (overpacking of the spheres).

Thus we have examined the features of deformation of a two-phase medium consisting of contacting elastic particles submerged in a compressible liquid. The absence of any effect of grain diameter on deformation is connected to the spatial averaging performed and the use of the basic approximations of the mechanics of inhomogeneous media [7]. The amount of deformation of the given medium is affected significantly by its structure (particle size and form, geometry of gaps, etc.). This structure determines the form of Eqs. (2.8), (2.9). In particular, calculation of the change in porosity is possible only for some fixed ordered structure.

Moreover, deformation in the particle contact region produces the dominant contribution to the character of the nonlinear overall deformation of the medium. With a number of contacts corresponding to cubical packing, the value of the deformation of a granular sand medium saturated by water is determined by pore pressure.

LITERATURE CITED

1. V. M. Dobrynin, Deformation and Change in Physical Properties of Petroleum and Gas Collectors [in Russian], Nedra, Moscow (1970).
2. H. A. Brandt, "Study of the speed of sound in porous granular media," J. Appl. Mech., 22, No. 4 (1955).
3. S. V. Kuznetsov, "A model of a porous soil (geometric parameters and filtration coefficient)," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1981).
4. R. S. Shelyapin and V. F. Chernyaev, "Some questions of sandy soil pressure on a fixed supporting wall using the theory of a granular medium," in: Foundations, Supports, and Soil Mechanics. Materials of the III All-Union Conference [in Russian], Budivel'nik, Kiev (1971).

5. K. Walton, "The effective elastic moduli of model sediments," Geophys. J. Roy. Astron. Soc., 43, No. 2 (1975).
6. M. A. Gol'dshtik, A. V. Lebedev, and V. N. Sorokin, "The valve effect in a granular layer," Inzh. Fiz. Zh., 34 (1978).
7. B. N. Nikolaevskii, K. S. Basniev, A. T. Gorbunov, and G. A. Zotov, Mechanics of Saturated Porous Media [in Russian], Nedra, Moscow (1970).
8. Ya. I. Frenkel', "On a theory of seismic and seismoelectric phenomena in moist soil," Izv. Akad. Nauk SSSR, Geogr. Geofiz., 8, No. 4 (1944).
9. A. I. Lur'e, Theory of Elasticity [in Russian], Nauka, Moscow (1970).
10. I. Fatt, "The Biot–Willis elastic coefficients for sandstone," J. Appl. Mech., 26, No. 28 (1959).

EQUATIONS OF MOTION OF GRANULAR MEDIA

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Acoustical studies of granular media used in petroleum and gas collectors have recently uncovered a number of unusual phenomena. Thus in highly porous bodies with empty or gas-saturated voids, the ratio of the velocities of S and P waves is often inexplicably large ($V_S/V_P > 1/\sqrt{2}$), which corresponds formally to negative values of the Poisson coefficient. According to data of [1] and other studies the value of V_S/V_P sometimes exceeds 0.75, i.e., the Poisson coefficient is less than −0.3. Moreover, wave velocities measured by various authors in the same specimens differ among themselves greatly (up to 10-15%), although in "good" test specimens (metallic ones, for example), they practically coincide. The experimental data indicate the insufficient development of physical theories of weak wave propagation in granular media such as hydrocarbon collectors.

Granular media possess two important unique features. First the linear dimensions of the grains allow introduction of a new dimensionless characteristic differing from the porosity f, which describes the pore space, namely $\eta = \sigma_0 r_0/3$, σ_0, the specific surface of the porous body, where r_0 is the mean grain radius. It has been proven by integral geometry that $0 \leqslant \eta \leqslant 1 - f$. Second, the presence of contacts between grains and sections of grains free from stress leads to a complex stressed state in each grain taken individually, so that aside from the mean (large scale) field, which changes markedly at distances of the order of a wavelength, a fluctuation field develops, which varies significantly at distance of the order of the individual grain size. Development of the fluctuation field leads to scattering of the energy contained in waves which are no longer purely P and S waves at each individual point, but only on the average. This implies that P and S waves are formed only by the average (large scale) stress and deformation fields, while fluctuations insure scattering of waves and a decrease in the amplitude of the mean field. In constructing a model of a continuous medium equivalent to a granular skeleton, the two features of the microinhomogeneous medium mentioned above must be considered. It is insufficient merely to require free equivalency of the media in the sense that the ratios of stress to deformation for the skeleton and the continuous models coincide. The presence of scattering and attenuation of the large scale field must lead to some wave "absorption" mechanism, produced by the scattering.

The above considerations demand a precise solution of the problem of elastic equilibrium for an individual grain, which in principle can be given by the ratio of stress to deformation at the center of the grain (i.e., the mean values of λ and μ in the structure) and the fraction of energy α contained in the fluctuation field referred to the mean field. These constants, which depend on the geometry of the pore space and material of the skeleton, allow transition to construction of an equation of motion of some set of particles with known mean values of the Lamé coefficients and known fraction of the energy scattered. It can be expected that the presence of isotropic scattering is equivalent to introduction of additional randomly oriented sources which collect the energy of the large scale field, attenuating the latter. The goal of the present study is to derive equations of motion (and equilibrium) for the mean field, since it is only this mean field which is recorded by any device utilizing

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